Solving (6) and (7) for $n = \pm 1$ yields the circularly polarized propagation constants for the modified TE_{11} mode. Then by use of (1), the saturation rotation can be obtained for the composite ferrite and dielectric structures shown in Fig. 1.

Some of these theoretical results are given in Fig. 2 and Fig. 3.

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Spectral Distribution of Thermal Noise in a Gas Discharge*

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Summary—**By means of thermodynamic considerations it is shown under which conditions microwave noise power generated by a gas discharge can be considered thermal. A critical analysis of Mumford's hypothesis is made.**

INTRODUCTION

MUMFORD [1] has pointed out that a fluorescent lamp filled with argon at 2-mm pressure and a drop of mercury having a saturated pressure of 6–8 microns, can be used as a standard microwave noise source having an equivalent temperature of 11,400°K. By considering the case of a black body radiating at a temperature of 11,400°K, and calculating from Wien's displacement law the wavelength of maximum radiation at this temperature, Mumford found $\lambda_m = 2535$Å. The fact that an intensive mercury line lies at 2537Å led Mumford to postulate that in a gaseous discharge, radiating light energy substantially at one particular wavelength λ, the microwave noise power available is the same as that available from a black body radiating maximum power at that wavelength. In this paper, it is shown under which conditions the noise power generated by a gas discharge can be considered thermal. To this end a critical analysis of Mumford's hypothesis is made. Three broad lines of attack are open.

1) A general quantitative formulation of the frequency spectrum distribution of the gas discharge from the far ultraviolet region down to the microwave range, on a kinetic theory basis. A theoretical treatment has been made by Parzen and Goldstein [2], but by invoking some critical arguments put forth by van der Ziel [3] it can be shown that their results hold only for microwave frequencies and would not hold for higher ones. The general treatment however is very difficult.

2) An experimental investigation.

3) An approach based on thermodynamical considerations which can be dealt with semiquantitatively. This course will be followed in this treatment.

THERMODYNAMIC THEORY OF THERMAL NOISE IN A GAS DISCHARGE

Mumford's hypothesis can be reformulated by dividing it into two parts.

1) In a gaseous discharge the microwave noise power available is the same as that available from a black body at a particular temperature, T.

2) That particular temperature, T, is of such magnitude as to correspond to a maximum power radiation at a wavelength λ_m at which light energy is radiated substantially monochromatically.

Consider a cavity with walls perfectly reflecting at all frequencies.

Let the cavity contain particles with Maxwellian distribution of velocities and in thermal equilibrium with radiation present in it. It has been shown by Einstein [4] that the radiation energy density in the cavity will then follow Planck's radiation law

$$U_\nu d\nu = (8\pi h\nu^3/c^3)[\exp(h\nu/\kappa T) - 1]^{-1} \cdot d\nu \qquad (1)$$

where T corresponds to the temperature of the particles.

Planck has shown [5], that if some carbon dust or any other material, capable of reaching high temperatures without absorbing too much heat, is brought into a perfectly reflecting cavity containing radiation, the radiation will redistribute itself and follow (1) after thermal equilibrium has been established. The temperature T will then be that of the material agent. Thus, if the cavity originally contained sharp mercury lines those would be absorbed, and partially re-emitted, and the radiation energy density would eventually obey (1).

Consider a gas discharge tube in a waveguide in a

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plane parallel to the electric vector and let both ends of the waveguide be terminated by a perfect short circuit. Let it be assumed that the cavity thus formed has perfectly reflecting walls at all frequencies in the spectrum under consideration. In this case, the material medium consists of the particles in the gas discharge. The electrons owing to the applied external field and their small mass have by far higher velocities than the other particles in the gas discharge. Since, on the other hand, the interaction energy between radiation and a charged particle is proportional to the velocity of the particle, the contribution of the electrons to the radiation energy will be much more important than that of any other particles in the gas discharge. If, in addition, the electron distribution is Maxwellian—and only Maxwellian, neglecting quantum effects—the concept of electron temperature T_e is meaningful and the radiation energy density in the cavity will follow (1) with

$$T = T_e. \tag{2}$$

It can be shown that the noise power available from a black body is given by the quantum theory form of Nyquist's theorem

$$dP = h\nu[\exp(h\nu/\kappa T) - 1]^{-1} \cdot d\nu. \tag{3}$$

In our case the power available using (2) is

$$dP = h\nu[\exp(h\nu/\kappa T_e) - 1]^{-1} \cdot d\nu. \tag{4}$$

So far it has been assumed that the walls were reflecting perfectly at all frequencies. In fact, perfect reflection obtains in the microwave region and therefore (4) holds in that frequency range.

For microwave frequencies one has

$$h\nu/\kappa T_e \ll 1 \tag{5}$$

so that (4) reduces to

$$dP = \kappa \cdot T_e \cdot d\nu. \tag{6}$$

The first part of Mumford's hypothesis is then valid. It is seen from (4) that the frequency corresponding to maximum power radiation is a function of the electron temperature T_e. On the other hand (4) holds irrespective of the particular properties associated with a given material in the cavity. This is a direct result of the second law of thermodynamics. It therefore follows that part two of Mumford's hypothesis is not valid.

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Design of Aperture-Coupled Filters*

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Summary—A procedure for the design of aperture-coupled filters is presented, based on the theory of conventional coupled circuits. This design procedure accounts for the relatively low insertion loss of aperture-coupled filters as compared with other known designs of microwave filters. The factors which contribute to this low value of insertion loss are the following:

1) Use of a high Q-mode configuration such as a cylindrical cavity in the TE_{011} mode.

2) Aperture coupling of elements eliminating the losses of impedance transforming sections of transmission line.

3) A mechanical design which eliminates joints at critical points and also provides control over interior surface finishes.

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I. INTRODUCTION

MANY system applications require band-pass filters at microwave frequencies having a low insertion loss. While band-pass filters can be built by cascading resonant cavities formed by irises in rectangular waveguide such as described by Mumford,[1] the insertion loss of this filter structure becomes prohibitively large for many applications at X band and higher frequencies.

[1] W. W. Mumford, "Maximally flat filters in waveguide," *Bell Sys. Tech. J.*, vol. 27, pp. 684–713; October, 1948.